Mass Defect, Nuclear Binding Energy, NBE – Curve and Magic Number

(Ref. AKD & RPS)

Mass defect: After discovery of neutron, it was observed that theoretically calculated atomic mass is always greater than that of experimentally measured atomic mass. This mass difference is known as mass defect of that atom. The mass defect (Δm) of an atom like $^{A}{}_{Z}X$ may be calculated as:

$(\Delta m) = [Zm_p + Zm_e + (A-Z)m_n – M]$ amu

or, $(\Delta m) = [Zm_H + (A-Z)m_n – M]$ amu

Where, M is the experimentally measured atomic mass and other terms are its usual meaning. Mass of an atom is measure in atomic mass unit (amu) and hence mass defect is also expressed in amu unit. Again from the Einstein's theory of special relativity we know that any mass loss is converted to equivalent amount of energy (according to the relation $E= MC^2$). So, mass loss of an atom always converted to an equivalent amount of energy which is stabilises the atom. This energy is known as nuclear binding energy. The nuclear binding energy (NBE) per nucleon of an atom is a measure of nuclear stability. Greater the value of NBE per nucleon, more stable will be the nucleus.

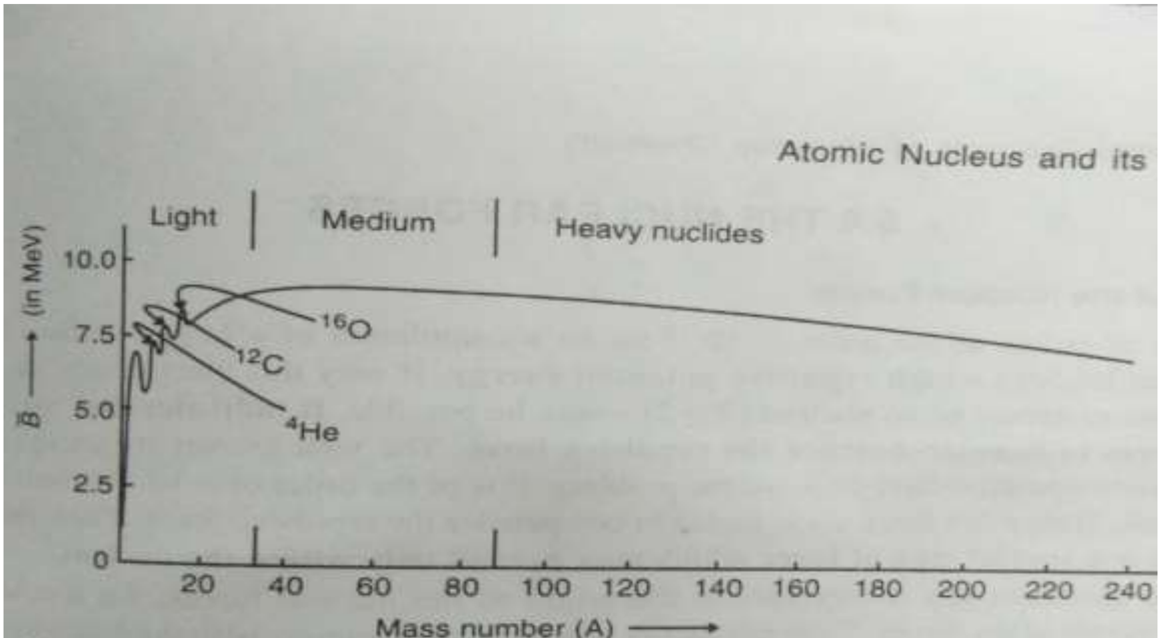
Again it is found that, 931 MeV amount of NBE is generated when mass defect is 1 amu.

So, in general, total NBE may be calculated as: NBE = Δm x 931 MeV and

NBE per nucleon ($\bar{B}$) = $\frac{\Delta \text{m x } 931}{A}$ MeV, where A is the mass number of the atom.

Therefore, it may be concluded that the nuclear binding energy of a nucleus may be considered to be the amount of energy released during the formation of the nucleus from its constituent neutrons and protons. On the other hand it is also the energy required to split up the nucleus into constituent neutrons and protons.

Nuclear Binding Energy Curve: If nuclear binding energy per nucleon ($\bar{B}$) is plotted against mass number of atoms (A), a very significant and almost smooth curve is obtained. The characteristic observations of the nuclear binding energy curve in three mass number region are depicted below:



Variation of nuclear binding energy per nucleon ($\bar{B}$) with mass number (A).

(i)Light nuclides (A< 30): From the binding energy curve, it is found that, in the lower mass number region the average binding energy increases rapidly and the maxima of $\bar{B}$ vary periodically for the values of A which are the integral multiples of four. These nuclides are: $^{4}_{2}He$, $^{12}_{6}C$, $^{16}_{8}O$, $^{20}_{10}Ne$, $^{24}_{12}Mg$ and $^{28}_{14}Si$ which are very much stable. In this group, only $^{8}_{4}Be$ is an exceptional one and it actually disintegrates into two alpha particles ($^{8}_{4}Be \rightarrow 2\ ^{4}_{2}He$).Thus in this region , the nucleus constructed by the alpha-particles as the blocking units are stabler compared to the neighbouring ones (Oddo-rule).

(ii) Medium nuclides (A =30-90): There is a rise in average nuclear binding energy ($\bar{B}$) from 8.0 MeV (for A=16) to~8.30 MeV (for A=28-32) and it remains almost constant ($\bar{B}$ = 8.5$\pm$ 0.2) up to A = 90. The region of transition metals (e.g. Fe, Co, Ni... etc.) at around A= 50-60 lies at the centre of the plateau. This range (A= 30-90) characterized by average binding energy 8.5$\pm$ 0.2 MeV gives the largest zone of the most stability . The value $\bar{B}$ = 8.5$\pm$ 0.2 is considered to be a diagnostic parameter in predicting the stability of the nuclides and these are most abundant in nature. The nuclides having $\bar{B}$ < 8.5$\pm$ 0.2 are considered to relatively unstable.

(iii) Heavy nuclides (A >90): After zirconium, $\overline{B}$ falls down gradually from~8.7 MeV to~7.5 MeV at A = 210. Up to A = 210, the nuclides are stable but after this limit, the nuclides become very much unstable. In fact, $^{209}_{83}Bi$ is the heaviest nuclide known to be stable.

It can be concluded that the stable nuclides (A > 30) possess $\overline{B}$ in the range 7.5 – 8.5 MeV .The nuclides (A >30) having $\overline{B}$ less than 7.5 MeV are radioactive.

Special characteristic features of NBE curve (basis of nuclear fission and fusion reaction):

The overall shape of B.E. –curve represents an interesting feature. With nuclei of small mass numbers, B.E. first increases with increase in mass number. This suggests the energy will be released if one could fuse lighter nuclei into the more stable ones. This is the basis of nuclear fusion reaction. On the other hand, in the high mass number region (say A>100), however, B.E decreases sharply with mass number, showing that the heavier nuclei are less stable. Here, energy would also be released if one could split such heavier nuclei into some lighter, stable fragments. This is the basis of nuclear fission reactions.

Another special feature of NBE curve is that, the nuclear binding energy curve has got a theoretical basis in the nuclear liquid drop model..

Magic Number: From the distribution as well as abundance of stable isotopes, it has been observed that nuclei containing some specific number of neutrons or protons are exceptionally stable and rich in isotopes. These numbers are **2, 8, 20, 50, 82 and 126**.They has been called magic numbers. The stability of nuclides containing either neutrons or protons equal to any magic number may be interpreted as due to having filled shell in them which can be explain from the nuclear shell model.

Example: (i) Elements having the largest number of stable isotopes contain 20 and 50 protons or 20, 50 or 82 neutrons. Tin (50 proton) has the maximum number of isotopes (ten). Calcium, with 20 protons, has six stable isotopes.

(ii) From the NBE-curve, it has been also observed that nuclides containing magic number of either proton or neutrons or both are more abundant than others in the same mass number region

($^{88}_{38}Sr$, $^{89}_{39}Y$,$^{90}_{40}Zr$ – stable isotopes, all containing 50 neutrons).