4. (a) In a sample of pitchblende the ratio of $^{206}Pb : ^{238}U = 0.2:1$ by weight. Calculate the age of the mineral, if half life of $^{238}U$ is $4.5 \times 10^9$ years. (all lead originated from Uranium).
   (b)Which one is more stable between $^{7}Li$ and $^{7}Be$? Predict the mode of decay of the unstable nuclide. [Given: $m_{7Be} = 7.01693u$  $m_{7Li} = 7.01600u$ , $m_e = 0.00055u$ ]

***(a) In a sample of pitchblende the ratio of $^{206}Pb : ^{238}U = 0.2:1$ by weight. Calculate the age of the mineral, if half life of $^{238}U$ is $4.5 \times 10^9$ years. (all lead originated from Uranium).*** ***3***

Solution: We let, there is 0.2x gm $^{206}Pb$ and 1x gm $^{238}U$ was present in pitchblende sample.

We know 206 gm $^{206}Pb$ is formed from 238 gm of $^{238}U$

So, 0.2x gm $^{206}Pb$ is generated from 238/206 x 0.2x gm = 0.231x gm of $^{238}U$

So, initially amount of $^{238}U$ was (1x+ 0.231x) = 1.231x gm

So, we can say that initial amount of $^{238}U$ ($N_0$) = 1.231x g and final amount of $^{238}U(N_t)$ =1x g

Again, we know that $\lambda=0.693/t_{1/2}$ where $\lambda$ is the disintegration constant and $t_{1/2}$ is half life time of the radioactive atom. So, $\lambda = 0.693/ 4.5 \times 10^9$ year$^{-1}$ = $1.54 \times 10^{-10}$ year$^{-1}$

Now, age of mineral (t) may be calculated as: $t = \frac{1}{\lambda} \log\frac{N_0}{N_t}$ where, $\lambda$ is the disintegration constant and $N_0$ and $N_t$ are initial and final number of radioactive atom present respectively. Here $N_0$& $N_t$ proportional to amount of $^{238}U$ present in gm.

or, $t = \frac{2.303}{1.54x\ 10^{-10}} \log \frac{1.231x}{1x}$ years = $1.4955 \times 10^{10} \times 0.09029$ years

or, $t = 0.134 \times 10^{10}$ years = $1.34 \times 10^9$ years

*So the age of the Pitchblende mineral = $1.34 \times 10^9$ years*

**Alternative Method:**

$\frac{weight\ of\ 206Pb}{weight\ of\ 238U} = \frac{0.2}{1} = \frac{2}{10}$

Here, mole of $^{206}$Pb = $\frac{weight\ of\ 206Pb}{Atomic\ weight\ of\ 206Pb}$ , similarly mole of 238U = $\frac{weight\ of\ 238U}{Atomic\ weight\ of\ 238U}$

So, $\frac{Mole\ of\ 206Pb}{Mole\ of\ 238U} = \frac{weight\ of\ 206Pb}{weight\ of\ 238U}$ x $\frac{Atomic\ weight\ of\ 238U}{Atomic\ weight\ of\ 206Pb} = \frac{weight\ of\ 206Pb}{weight\ of\ 238U}\ x\ \frac{238}{206} = \frac{2}{10}$ x $\frac{238}{206} =$ $\frac{\mathbf{0.231}}{\mathbf{1}}$

So, we can say mole of $^{206}$Pb = 0.231 and mole of $^{238}$U = 1.000

So, at the zero time i.e., when disintegration of $^{238}$U started, mole of Uranium -238 present in Pitchblende sample = 1.000+0.231 = 1.231

So, $N_0$ = 1.231 and $N_t$ = 1.000

We know t = $\frac{1}{\lambda}$ log$\frac{N_0}{Nt}$ where, $\lambda$ is the disintegration constant and $N_0$ and $N_t$ are initial and final number of radioactive atom present respectively. Here $N_0$& $N_t$ proportional to mole of $^{238}$U.

So, t = $\frac{2.303}{1.54x\ 10^{-10}}$ log $\frac{1.231}{1}$ = 1.34 x $10^9$ years

*So the age of the Pitchblende mineral = 1.34 x $10^9$ years*

***(b) Which one is more stable between $^7$Li and $^7$Be? Predict the mode of decay of the unstable nuclide. [Given: $m_{7Be}$ = 7.01693u    $m_{7Li}$ = 7.01600u , $m_e$ = 0.00055u ]             3***

Since $^7$Be has a larger mass (7.01693u) than $^7$Li (7.01600u), the former can decay spontaneously into $^7$Li but not vice versa.

$^7_4Be + {}_{-1}e$ (K-electron capture) $\rightarrow$ $^7_3Li$

If n/p ratio of a nucleus is lower than that of the particular stability range of that atom, then that nucleus will be unstable and must disintegrate to form a stable nucleus of a new atom. There are two types of decay process in which Z is decreased by one unit without a change in mass number (A) viz. positron ($\beta^+$) emission and K-electron capture. These two processes have different mass balance requirements.

***Actually positron emission occurs only if the nuclidic mass of the parent species exceeds the nuclidic mass of the daughter by at least twice the rest mass of the electron i.e. 2 x 0.00055u = 0.00110 u*** **.In the present case the actual mass difference between parent and daughter nuclides is 7.01693- 7.01600 = 0.00093u. So, positron emission in this case is impossible and $^7$Be undergoes K-capture into $^7$Li.**