James Clerk Maxwell in 1800s mathematically established that light is an electromagnetic wave.

Much later the photonic nature of light was established by Einstein.

Light propagates through space in a wave like manner & shows particle like behaviour during the processes of emission and absorption.

Maxwell's eqn.s, in free space, describing the relation b/w the electric & magnetic fields are,

$$\bar{\nabla} \cdot \bar{E} = 0,$$

$$\bar{\nabla} \cdot \bar{B} = 0,$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t},$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}:$$

In integral form,

$$\oint_C \bar{E} \cdot d\bar{l} = -\iint_A \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

$$\oint_C \bar{B} \cdot d\bar{l} = \mu_0 \epsilon_0 \iint_A \frac{\partial \bar{E}}{\partial t} \cdot d\bar{S}$$

$$\oiint_A \bar{B} \cdot d\bar{S} = 0$$

$$\oiint_A \bar{E} \cdot d\bar{S} = 0$$

$A \rightarrow$ open area

$C \rightarrow$ closed curve surrounding the borders of A.

$d\bar{S} \rightarrow$ outward drawn vector area on A.

Maxwell's eqns are coupled 1st order partial differential eqns for $\overline{E}$ & $\overline{B}$.

Decoupling the eqns would give,

$$\nabla^2 \overline{E} = \mu_0 \epsilon_0 \frac{\partial^2 \overline{E}}{\partial t^2}$$

$$\nabla^2 \overline{B} = \mu_0 \epsilon_0 \frac{\partial^2 \overline{B}}{\partial t^2}$$

which are 2nd order PDEs.

In vacuum, each Cartesian component of $\overline{E}$ & $\overline{B}$ satisfies the 3D wave eqn,

$$\nabla^2 f = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

where $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \ m/s$

is the velocity of light in free space.

Light is a transverse EM wave.
There can be three types of polarization for an em wave:

1. Linearly polarized or plane polarized.
2. Circularly polarized.
3. Elliptically polarized.

If a string whose one end is fixed, is moved up & down, then a transverse wave is generated. Each point on the string executes sinusoidal oscillation in a straight line (say, along x-axis). This wave is linearly polarized or plane polarized as the string is always confined to the $x-z$ plane.

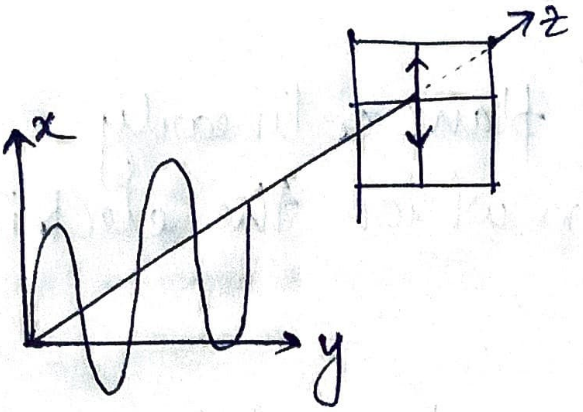
The displacement for a linearly polarized wave is given by the form,

$$x(z,t) = a\cos(kz - \omega t + \phi_1) \quad -(1)$$

$$\& \; y(z,t) = 0$$

Any arbitrary point $z = z_0$ will execute a SHM of amplitude a. The string can also be made to oscillate in the $y-z$ plane,

$$y(z,t) = a\cos(kz - \omega t + \phi_2) \quad \& \quad x(z,t) = 0 \quad -(2)$$

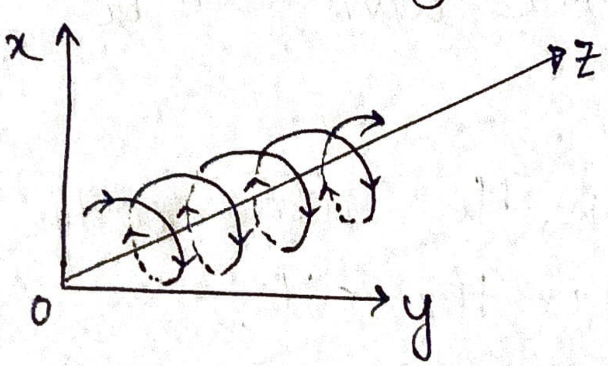


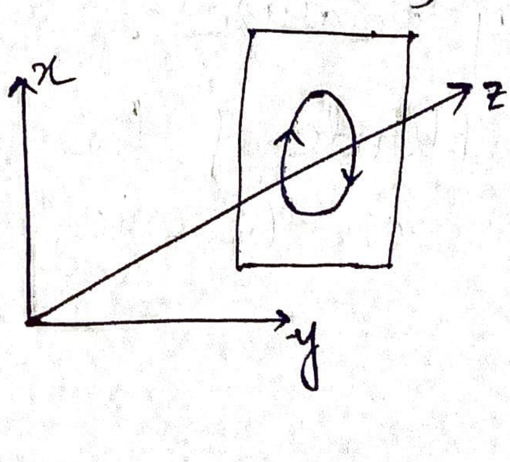
The string can be made to vibrate in any plane containing the z-axis.

If one rotates the end of the string on the circumference of a circle then each point of the string will move in a circular path.
Then such a wave is known as a circularly polarized wave.
Displacement in this case is given by,

$$\left.\begin{aligned} x(z,t) &= a\cos(kz - \omega t + \phi) \\ y(z,t) &= a\sin(kz - \omega t + \phi) \end{aligned}\right\} \quad (3)$$

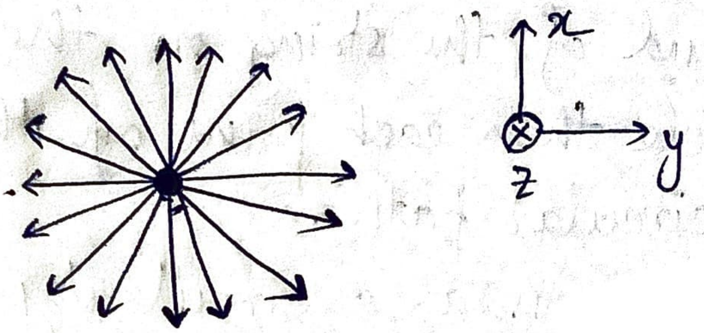
so that $x^2 + y^2 = a^2$ is a constant (eqn. of a circle of radius a.)





The plane of vibration for a plane or linearly polarized light is the plane in which the electric field resides.

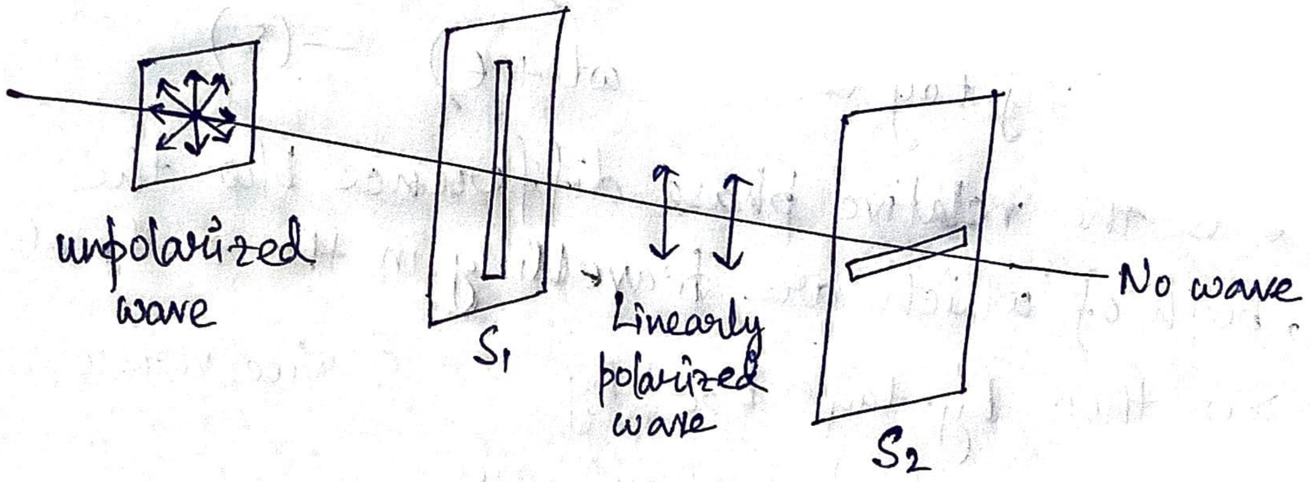
If this plane of vibration is changed in a random manner in very short intervals of time, i.e. short compared to the detection time, then such a wave is known as an unpolarized wave.



If an ~~a~~ unpolarized light is incident on a slit S_1 then the displacement associated with the transmitted wave will be along the length of the slit. The plane of polarization of the transmitted wave will depend on the orientation of the slit, & the transmitted wave will be linearly polarized.

The amplitude of the transmitted wave will not depend on the orientation of the slit.

If this linearly polarized wave is allowed to pass through another slit S_2, then the intensity of the emerging wave will depend on the relative orientation of S_2 wrt S_1.



$S_1 \longrightarrow$ polarized

$S_2 \longrightarrow$ analyzer

The plane of vibration contains both $\bar{E}$ and $\bar{k}$, the electric field vector & the propagation vector $\bar{k}$ in the direction of motion.

Two harmonic, linearly polarized lightwaves of the same frequency & moving through the same region of space in the same direction will combine to to form a linearly polarized wave, if the elec. fields vectors are collinear.

For ~~at~~ orthogonal elec. field vectors, the resultant wave may or may not be linearly polarized.

LINEAR POLARIZATION

Consider two linearly polarized light waves whose elec. field vectors are mutually orthogonal,

$$\bar{E}_x(z,t) = \hat{i} E_{0x} \cos(kz - \omega t) \quad -(4)$$

&, $$\bar{E}_y(z,y t) = \hat{j} E_{0y} \cos(kz - \omega t + \epsilon) \quad -(5)$$

where ϵ is the relative phase difference b/w the waves, both of which are travelling in the z-direction

If $\epsilon > 0$, then E_y lags E_x by ϵ, & vice versa.

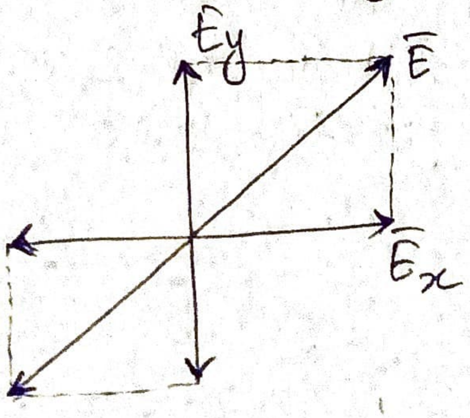
The resultant optical disturbance is,

$$\vec{E}(z,t) = \bar{E}_x(z,t) + \bar{E}_y(z,t) \quad -(6)$$

For $\epsilon = 0$ or an integral multiple of $\pm 2\pi$, the waves are in-phase.

$$\therefore \bar{E} = (\hat{i} E_{0x} + \hat{j} E_{0y}) \cos(kz - \omega t) \quad -(7)$$

The resultant wave has a fixed amplitude of $(\hat{i} E_{0x} + \hat{j} E_{0y})$, hence it is also linearly polarized.



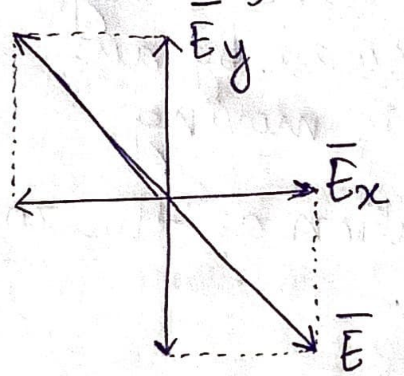
Viewed on a plane of observation, the $\vec{E}$ field progresses through one complete oscillatory cycle as the wave advances along the z-axis through one wavelength.

We can also resolve any plane-polarized wave into two orthogonal components.

For ϵ be an odd integer multiple of $\pm\pi$, the two waves are 180° out-of-phase, and

$$\bar{E} = (\hat{i}E_{0x} - \hat{j}E_{0y})\cos(kz - \omega t) \quad -(8)$$

The above is again linearly polarized, but the plane of vibration has been rotated (not necessarily by 90°) from that of the previous condition.



CIRCULAR POLARIZATION

Let two linearly polarized light waves whose elec. field vectors are perpendicular and both the constituent waves have equal amplitudes,

$E_{0x} = E_{0y} = E_0$

and their relative phase difference

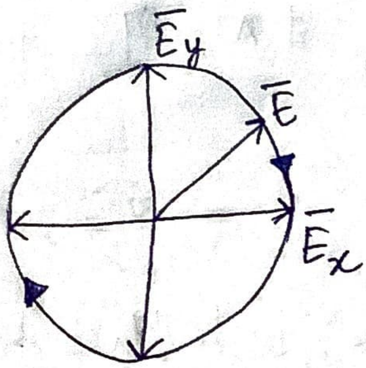
$\epsilon = -\frac{\pi}{2} + 2m\pi$, where $m = 0, \pm 1, \pm 2$.

Then $\bar{E}_x(z,t) = \hat{i}E_0\cos(kz - \omega t) \quad -(9)$

& $\bar{E}_y(z,t) = \hat{j}E_0\sin(kz - \omega t) \quad -(10)$

& the resultant wave is,

$$\bar{E} = E_0\left[\hat{i}\cos(kz-\omega t) + \hat{j}\sin(kz-\omega t)\right] \quad -(11)$$



It is an elec. field with const. amplitude but rotating clockwise with the ~~same~~ freq. ω with which it oscillates.

$\bar{E}$ is no longer restricted to a single plane.

Such a wave is said to be right circularly polarized. The elec. field is viewed by an observer toward whom the wave is moving.

$\bar{E}$-vector makes one complete rotation as the wave advances through one wavelength.

Again, if $\epsilon = \frac{\pi}{2} + 2m\pi$, where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$

then

$$\bar{E} = E_0\left[\hat{i}\cos(kz-\omega t) - \hat{j}\sin(kz-\omega t)\right] \quad -(12)$$

In this case, the amplitude is still E_0 but $\bar{E}$-vector now rotates counterclockwise ~~be~~ when viewed by an observer toward whom the wave is moving.

The wave is said to be left circularly polarized.

If we add a right circular wave (11) & a left circular wave (12), we get

$$\vec{E} = 2E_0 \hat{i} \cos(kz - \omega t) \quad \text{— (13)}$$

which has a const. amplitude of $2E_0$ & it varies along x direction

Thus, (13) is a linearly polarized wave.

Two oppositely polarized circular waves of equal amplitudes combine to form a linearly polarized wave.

ELLIPTICAL POLARIZATION:

Both linear & circularly polarized light may be considered ~~as~~ to be special cases of elliptically polarized light.

In case of elliptical polarization, the resultant elec. vector $\bar{E}$ will rotate, and change its magnitude, as well. The endpoint of the vector traces out an ellipse, in a plane $\perp$ to $\bar{k}$ as the wave moves forward.

The components of the two orthogonal electric field vectors are,

$$E_x = E_{0x} \cos(kz - \omega t) \quad \text{— (14)}$$

$$E_y = E_{0y} \cos(kz - \omega t + \epsilon) \quad \text{— (15)}$$

Then, $\frac{E_y}{E_{oy}} = \cos(kz-\omega t)\cos\epsilon - \sin(kz-\omega t)\sin\epsilon$

Also, $\frac{E_x}{E_{ox}} = \cos(kz-\omega t)$

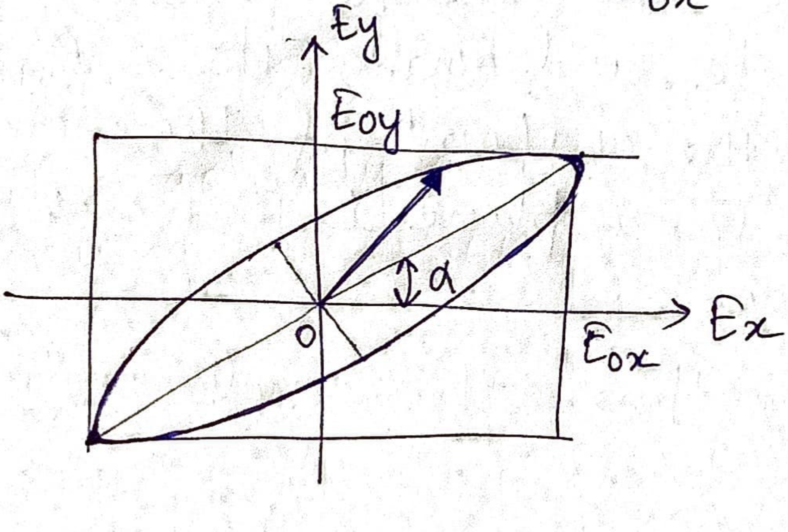
$\therefore \frac{E_y}{E_{oy}} = \frac{E_x}{E_{ox}}\cos\epsilon - \sqrt{1-\left(\frac{E_x}{E_{ox}}\right)^2}\,.\sin\epsilon$

or, $\left(\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}}\cos\epsilon\right)^2 = \left[1-\left(\frac{E_x}{E_{ox}}\right)^2\right]\sin^2\epsilon$

or, $\left(\frac{E_y}{E_{oy}}\right)^2 + \left(\frac{E_x}{E_{ox}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right)\cos\epsilon = \sin^2\epsilon$ — (16)

The above is an eqn of an ellipse making an angle α with the (E_x, E_y) coord. system, such that,

$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\epsilon}{E_{ox}^2 - E_{oy}^2} \quad \text{— (17)}$$



For $\alpha = 0$ or $\epsilon = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$ the principal axes will align with the coord. axes.

Then (16) reduces to, $\frac{E_y^2}{E_{oy}^2} + \frac{E_x^2}{E_{ox}^2} = 1$ — (18)

If $E_{oy} = E_{ox} = E_o$, then we get,

$$E_y^2 + E_x^2 = E_o^2 \quad \text{— (19)}$$

which is a circle (circularly polarized light).

For $\epsilon = 2m\pi$ where $m = 0, 1, 2, 3, \ldots,$

$$E_y = \frac{E_{oy}}{E_{ox}} E_x \quad \text{— (20)}$$

which is a st. line having a +ve slope of $\left(\frac{E_{oy}}{E_{ox}}\right)$

Similarly for $\epsilon = (2m+1)\pi$ where $m = 0, 1, 2, 3, \ldots$

$$E_y = -\frac{E_{oy}}{E_{ox}} E_x \quad \text{— (21)}$$

which is also a st. line with a -ve slope $-(E_{oy}/E_{ox})$.

So we have linearly polarized light in above two cases.

A linear/plane polarized light $\rightarrow$ P-state.

Left circular light $\rightarrow$ L state.

Right " " $\rightarrow$ R state.

Elliptically polarized light $\rightarrow$ E - state.

Natural Light :

An ordinary light source consists of a very large no. of randomly oriented atomic scale scatteres. Each ~~exit~~ excited atom radiates a polarized wavetrain for $\sim 10^{-8}$ s.

Now all emissions having the same freq. will combine to form a single resultant polarize wave, which persits for $\sim 10^{-8}$ s.

New wavetrains are constantly emitted, & the polarization of each are randomly oriented.

The changes in polarization are completely unpredictable & ~~if~~ these are so rapid that it is impossible to differentiate out any particular state of polarization.

The wave is referred to as natural light or unpolarized light.

It is actually a light composed of rapidly varying succession of different polarization states.

More often, whether natural or artificial in origin, light is neither completely polarized nor completely ~~unplo~~ unpolarized & is partially polarized.