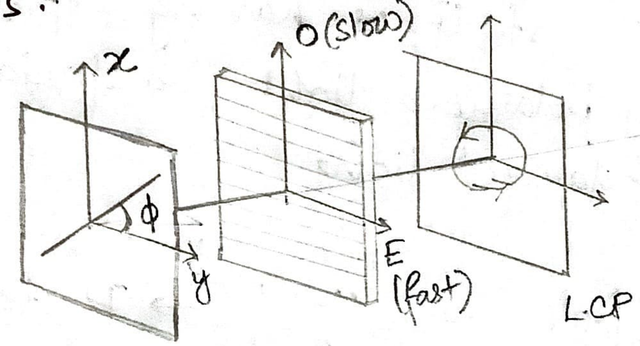
INTERFERENCE OF POLARIZED LIGHT :

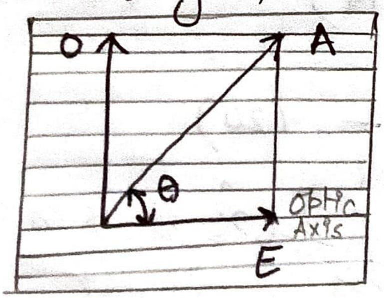
Let a plane-polarized beam of light from a nicol is incident normally on a thin plate of calcite crystal cut with its faces parallel to the optic axis.



We assume the y-axis be along the optic axis.
If the incident beam is x-polarized, the beam will propagate as an O-wave through the calcite & the E-wave will be absent.

If the incident beam is y-polarized, the beam will propagate as an E wave & the O wave will be absent.

For any other state of polarization of the incident beam, both the O & E components will be present. The E-waves having vibrations parallel to the optic axis will travel faster than the O-wave but along the same path.



For a -ve crystal, $n_e < n_o$ & E-wave will travel faster than O-wave. So the direction parallel to the optic axis in a -ve crystal are often called the fast axis & the directions perpendicular to it are called the slow axes.

For a +ve crystal, $n_e > n_o$, so the oppo. of above is true.

Let the elec. field vector of amplitude E_0 associated with the incident polarized beam make an angle ϕ with the y-axis.

This beam can be broken up into two linearly polarized beams (vibrating in phase), polarized along x- & y-directions with amplitudes,

$O = E_0 \sin\phi$ & $E = E_0 \cos\phi$, respectively.

The x-component ($E_0 \sin\phi$) passes through as an O-wave with velocity $\frac{c}{n_o}$ & the y-component ($E_0 \cos\phi$) passes through as an E-wave with velo. $\frac{c}{n_e}$.

Since $n_e \neq n_o$, the two beams will propagate with diff. velocities & when they come out of the crystal, they, in general, will not be in phase.

The emergent beam will be a superposition of these two beams &, in general, will be elliptically polarized.

The incident beam (plane-polarized) be given by $E_0 \cos(kz - \omega t)$.

Then the x & y components of the beam are

$$\left.\begin{aligned} E_x &= E_0 \sin\phi \cos(kz - \omega t), \text{ and} \\ E_y &= E_0 \cos\phi \cos(kz - \omega t) \end{aligned}\right\} - (1)$$

where $k = \omega/c$ is the wave number in free space

Let the surface of the crystal be $z=0$ plane

Then, $$\left.\begin{aligned} E_x(z=0) &= E_0 \sin\phi \cos\omega t \\ \& \ E_y(z=0) &= E_0 \cos\phi \cos\omega t \end{aligned}\right\} (2)$$

Inside the crystal $(z \neq 0)$ & the two components are given by,

$$\left.\begin{aligned} E_x &= E_0 \sin\phi \cos(n_o kz - \omega t) \quad [\text{O-wave}] \\ E_y &= E_0 \cos\phi \cos(n_e kz - \omega t) \quad [\text{E-wave}] \end{aligned}\right\} (3)$$

If the thickness of the calcite be d, then at the emerging surface the components are,

$$\left.\begin{aligned} E_x &= E_0 \sin\phi \cos(n_o kd - \omega t) \\ E_y &= E_0 \cos\phi \cos(n_e kd - \omega t) \end{aligned}\right\} - (4)$$

By appropriately choosing the instant $t=0$ we can write,

$$\left.\begin{aligned} E_x &= E_0 \sin\phi \cos(\omega t - \theta) \\ E_y &= E_0 \sin\phi \cos\omega t \end{aligned}\right\} - (5)$$

where $\theta = kd(n_o - n_e) = \frac{\omega}{c}(n_o - n_e)d = \frac{2\pi}{\lambda}(n_o - n_e)d$ — (6)

is the phase difference b/w the ordinary & the extraordinary beams.

θ depends on the thickness of the crystal.

For $\theta = 0, 2\pi, 4\pi, \ldots$, the emergent vibration will remained unchanged as the incident vibration.

If d is such that $\theta = \pi, 3\pi, 5\pi, \ldots$, we get linear vibration making angle 2ϕ with its original direction.

For all intermediate θ values the emergent vibration is in an ellipse, the shape of which is determined by the θ & ϕ values.

* For a crystal to produce circularly polarized light, the amplitudes of O & E rays must be equal, & the phase difference $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

A crystal is said to be a Quarter Wave Plate (QWP) or $\lambda/4$ plate if the thickness is so adjusted so as to introduce $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ phase change b/w the O & E vibrations (which corresponds to a path diff. of a quarter of a wavelength, $\lambda/4$).

When a QWP is oriented such that the optic axis makes 45° with the plane of the incident polarized light, the emergent light is circularly polarized as then O & E rays have equal amplitudes.

$\therefore \phi = \frac{\pi}{4}$ & $\theta = \frac{\pi}{2}$

Thus, for the emergent beam, using (5),

$$\left.\begin{aligned} E_x &= E_0 \sin\frac{\pi}{4} \cdot \cos\left(\omega t - \frac{\pi}{2}\right) = \frac{E_0}{\sqrt{2}} \sin\omega t \\ E_y &= E_0 \cos\frac{\pi}{4} \cos\omega t = \frac{E_0}{\sqrt{2}} \cos\omega t \end{aligned}\right\} \quad (7)$$

For a -ve crystal, calcite, $\theta = \frac{\pi}{2}$ will give LCP wave.

The thickness of the crystal should be then,

$$d = \frac{c}{\omega} \frac{\theta}{(n_o - n_e)} = \frac{c}{\omega} \cdot \frac{1}{(n_o - n_e)} \cdot \frac{\pi}{2} = \frac{\lambda}{4(n_o - n_e)} \quad -(8)$$

* If the thickness of the crystal is such that $\theta = \pi$, the crystal is said to be a Half wave Plate (HWP), which corresponds to a path diff. of half of a wavelength, $\lambda/2$.

This plate merely alters the direction of vibration of a linearly polarized light by an angle 2ϕ from the original direction where ϕ is the angle b/w the incident vibrations & the ~~optic axis~~ principal section.

Let $\phi = \frac{\pi}{4}$ & $\theta = \pi$ (x & y components have equal amplitudes)

Thus for the emergent beam,

$$E_x = E_0 \sin\frac{\pi}{4} \cos(\omega t - \pi) = -\frac{E_0}{\sqrt{2}} \cos\omega t.$$
$$E_y = E_0 \cos\frac{\pi}{4} \cos\omega t = \frac{E_0}{\sqrt{2}} \cos\omega t. \qquad (9)$$

The above gives a linearly polarized ~~by~~ beam on superposition whose direction of polarization is making an angle 2ϕ ($= 90^\circ$) with the original direction of vibration, i.e., 135° ($= 90^\circ + 45^\circ$) with the optic (y) - axis.

If this emergent beam is incident on a QWP made from calcite, the emergent beam will be RCP.

The thickness of the crystal will be then,

$$d = \frac{c}{\omega} \cdot \frac{\theta}{(n_o - n_e)} = \frac{c}{\omega} \cdot \frac{\pi}{(n_o - n_e)} = \frac{\lambda}{2(n_o - n_e)} \qquad -(10)$$

A LCP wave after propagating through a Calcite HWP gets converted to a RCP wave & vice-versa.

* If the thickness of a QWP is an odd multiple of (8), i.e., if

$$d = (2m+1) \cdot \frac{\lambda}{4(n_o - n_e)} \; ; \quad m = 0, 1, 2, \ldots \qquad -(11)$$

then, when $\phi = \pi/4$, the emergent beam will be LCP for $m = 0, 2, 4, \ldots$ & RCP for $m = 1, 3, 5, \ldots$

* For a HWP, the thickness of the plate (-ve) can be an odd multiple of (10), then

$$d = (2m+1) \cdot \frac{\lambda}{2(n_o - n_e)} \quad ; \quad m = 0, 1, 2, \dots \qquad -(12)$$

* If the crystal thickness is such that if $\theta \neq \pi/2, \pi, 3\pi/2, 2\pi, \dots$, the emergent beam will be elliptically polarized

Now for a +ve crystal (quartz), $n_e > n_o$, then the emergent beam can be expressed as,

$$\left. \begin{aligned} E_x &= E_0 \sin\phi \cos(\omega t + \theta') \\ E_y &= E_0 \cos\phi \cos\omega t \end{aligned} \right\} (13)$$

where $\theta' = \frac{\omega}{c} d (n_e - n_o)$ — (14)

Then for a QWP,

$$d = (2m+1) \cdot \frac{\lambda}{4(n_e - n_o)} \quad ; \quad m = 0, 1, 2, \dots \qquad -(15)$$

So, for a incident linearly polarized light at $\phi = \pi/4$, the emergent light is a RCP wave.

* If the thickness of the plate is such that $\theta = 2\pi$, the plate is said to be a Full Wave Plate since the corresponding path diff. is equal to one wavelength.

E & O waves are in phase & there will be no change in the direction of vibration of the incident beam.

Optical elements which are used to change the polarization of an incident wave are known as Retarders.

Eg: Full-wave plate or full-wave retarder,
HWP or half-wave retarder,
QWP or quarter-wave retarder.
