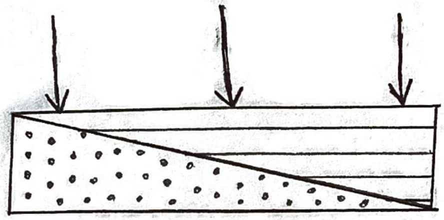
Compensators :

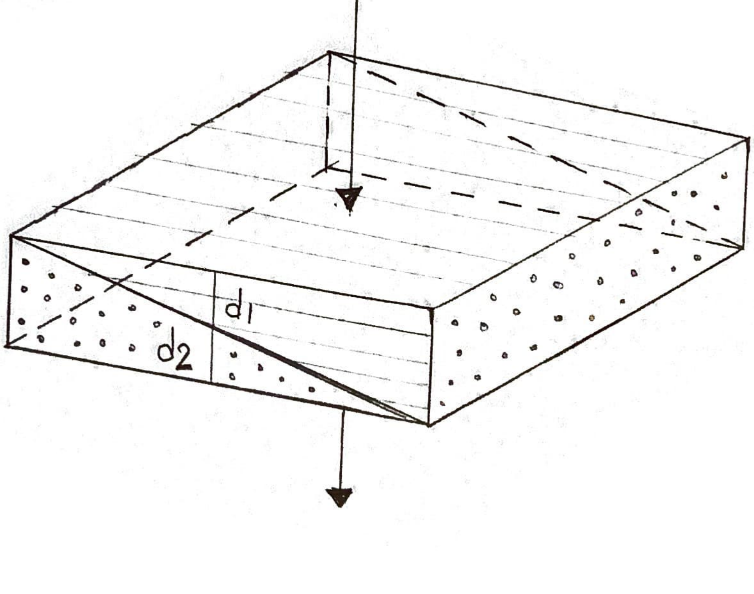
A compensator is an optical device that is capable of impressing a controllable retardance on a wave.

Unlike a wave plate, where the phase difference b/w the O & E waves is fixed, it can be varied continuously in a compensator.

Babinet Compensator :

Consists of two wedge-shaped prisms of calcite or quartz cut at a very small angle $^{\sim 2.5^\circ}$. The optic axes of the two prisms are parallel & perpendicular to the two refracting edges.





A ray passing vertically downboard through the device at some arbitrary point will traverse a thickness d_1 in the upper wedge & d_2 in the lower one.

Using eqn. (6) the phase diff. b/w the E & O waves imparted by the upper crystal is,

$$\theta_1 = \frac{2\pi}{\lambda}(|n_o - n_e|)d_1$$

& that due to the lower crystal is,

$$\theta_2 = -\frac{2\pi}{\lambda}(|n_o - n_e|)d_2$$

Then the total phase diff., or retardance, is

$$\theta = \theta_1 + \theta_2 = \frac{2\pi}{\lambda}(d_1 - d_2)(|n_o - n_e|)$$

If a plane polarized light is incident normally on the compensator with the plane of vibration at some arbitrary angle ϕ to the optic axis, it will be broken up into two components.

The E component parallel to the optic axis in the upper wedge travels faster (∵ calcite) than the O component until it reaches the 2nd wedge.

At this point the E vibrations become the O vibrations since it is now perpendicular to the optic axis of the 2nd wedge. At the same point the O vibrations from the 1st become the E vibration in the 2nd.

The two vibrations exchange velocities in passing from one prism to the other, so that the effect of one prism tends to cancel the effect of the other.

Along the centre at C, where $d_1 = d_2$, the cancellation is complete & the effect is that of a plate of zero thickness, i.e., $\theta = 0$, for all wavelengths.

On each side of C, one vibration will be behind or ahead of the other (depending on the crystal) because of the diff. path lengths.

If made of calcite (-ve), the $\mathcal{E}$ wave leads the O wave in the upper wedge, & O wave leads the $\mathcal{E}$ wave in the lower wedge.

$\therefore$ If $d_1 > d_2$, θ gives the total angle by which the $\mathcal{E}$ wave leads the O wave.

The converse is true for quartz (+ve).

The retardation will vary from point to point, being const. along the width of the compensator where the wedge thicknesses are themselves const.

If light ~~ono~~ enters through a slit parallel to ~~one of~~ the edge of the wedges & if the wedges are moved horizontally, we can get any θ

value b/w the two components.

ANALYSIS OF POLARIZED LIGHT :-

A polarized wave can be characterized by diff. states of polarizations,

→ linearly polarized,
→ circularly ,, ,
→ elliptically ,, ,
→ unpolarized,
→ mixture of linearly polarized & unpolarized,
→ ,, ,, circularly ,, & ,, , or
→ ,, ,, elliptically ,, & ,, light.

Placing a polaroid in the path of the beam of polarized light & rotating about the direction of propagation, the state of polarization of the incident light can be analyzed.

(a) If there is complete extinction at two positions of the polarizer, then the incident beam is linearly polarized.

(b) If there is no variation of intensity, then the incident beam is either unpolarized or circularly polarized or a mixture of unpolarized & circularly polarized light.

If a quarter wave plate is placed b/w the source of light & the polaroid, & there is no change in the intensity of the light, then the incident beam is unpolarized.

As QWP transforms a circularly polarized light into a linearly polarized one, so if there is complete extinction at two positions, then the incident beam is circulary polarized.

Again, if there is a varition of intensity, without complete extinction, then the beam is a mixture of unpolarized & circularly polarized light.

(c) If there is a variation of intensity, without complete extinction, then the beam is either elliptically polarized or a mixture of linearly polarized & unpolarized lights or a mixture of elliptically polarized & upolarized light.

If now a QWP is placed in front of polaroid with its optic axis parallel to the pass axis of the polaroid at the position of max. intensity. The elliptically polarized light will transform to a linearly polarized light. Thus, if two positions of the of the polaroid is obtained where there there is complete extinction, then the original beam is elliptically polarized.

If complete extinction does not occur, & the position of max. intensity occurs at the same orientation as before, the beam is a mixture of unpolarized & linearly polarized light.

If the position of max. intensity occurs at a diff. orientation of the polaroid, then the beam is a mixture of elliptically polarized & unpolarized light.

PROBLEMS

(1) A LCP beam ($\lambda = 5893\,\mathring{A}$) is incident normally on a calcite crystal (with its optic axis cut parallel to the surface) of thickness 0.005141 mm. What will be the state of polarization of the incident beam?

A LCP light at $z=0$ plane is represented by,

$$E_x = E_0 \sin\omega t \quad ; \quad E_y = E_0 \cos\omega t.$$

$$\theta = \frac{(n_0 - n_e)\, d \times 2\pi}{\lambda} = \frac{(1.65835 - 1.48640) \times 5.141 \times 10^{-6} \times 2\pi}{5.893 \times 10^{-7}}$$

$$\simeq 3\pi$$

$\therefore$ The emergent beam will be,

$$E_x = E_0 \sin(\omega t - \theta) = E_0 \sin(\omega t - 3\pi) = -E_0 \sin\omega t$$

$$E_y = E_0 \cos\omega t$$

The emergent beam thus represents a RCP beam.

(2) A LCP beam ($\lambda = 5893\,\mathring{A}$) is incident on a quartz crystal (with its optic axis cut parallel to the surface) of thickness 0.022 mm. Determine the state of polarization of the emergent beam. Assume n_o & n_e to be 1.54425 & 1.55336, respectively.

$E_x = E_0 \sin\omega t$; $E_y = E_0 \cos\omega t$. at $z=0$

$$\theta' = (n_e - n_o)\frac{2\pi}{\lambda}\cdot d = 2\pi \cdot \frac{0.00911 \times 2.2 \times 10^{-5}}{5.893 \times 10^{-7}} \approx 0.68\pi$$

The emergent beam,

$$E_x = E_0 \sin(\omega t + \theta') = E_0 \sin(\omega t + 0.68\pi)$$

$$E_y = E_0 \cos\omega t$$

The above represents an elliptically polarized light since, $\theta \neq \pi/2, \pi, 3\pi/2, 2\pi, \ldots$

(3) Calculate the thickness of quartz HWP for $\lambda = 5890$ Å. $n_o = 1.54424$ & $n_e = 1.55335$.

For +ve crystal (quartz),

$$\theta' = \frac{2\pi}{\lambda}\cdot d\,(n_e - n_o)$$

$\therefore$ For QWP, $d = \dfrac{\lambda}{2\pi}\cdot\dfrac{\pi}{2}\cdot\dfrac{1}{(n_e - n_o)}$

$$= \frac{5890 \times 10^{-10}\ \text{m}}{4(1.55335 - 1.54424)} \simeq 16.16\,\mu\text{m}$$

For HWP, $d = \dfrac{\lambda}{2\pi}\cdot\pi\cdot\dfrac{1}{(n_e - n_o)}$

$$= 32.33\,\mu\text{m}.$$

(4) What will be the Brewster angle for a glass slab ($n = 1.5$) immersed in water ($n = 4/3$)?

From Brewster's Law,

$$\tan \bar{\phi} = \frac{n_2}{n_1} = \frac{1.5}{4/3} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\bar{\phi} = \tan^{-1}\left(9/8\right) = 48.4^\circ$$